$$= \frac{-2-2i}{-1-\sqrt{2}+i} = \frac{2+2i}{(1+\sqrt{2})-i}$$

$$z_w = \frac{(2+2i)\left[(1+\sqrt{2})+i\right]}{(1+\sqrt{2})^2+1}$$

$$= \frac{2\,(1+\sqrt{2}+i+(1+\sqrt{2})i-1)}{4+2\sqrt{2}}$$

$$= \frac{\sqrt{2}+i(2+\sqrt{2})}{2+\sqrt{2}}$$

$$z_w = (\sqrt{2}-1)+i$$

b) $f = r_{(w,\frac{3\pi}{4})} \circ h_{(A,\sqrt{2})}$

f est une similitude directe de rapport $\sqrt{2}$ d'angle $\frac{3\pi}{4}+\pi = -\frac{\pi}{4}\ (2\pi)$

$$f_{(A)} = r_{(w,\frac{3\pi}{4})}(A) = A'$$

Avec $z_{A'} = e^{3i\frac{\pi}{4}}(z_A - z_w)+z_w$

$z_{A'} = \left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)(3-i-\sqrt{2}+1-i)+\sqrt{2}-1+i$

$z_{A'} = \frac{1}{\sqrt{2}}(-1+i)(4-\sqrt{2}-2i)+\sqrt{2}-1+i$

$z_{A'} = 3i\sqrt{2}$

- Soit I le centre de f

$$f_{(A)} = A' \iff \begin{cases} IA' = \sqrt{2}\,IA \\ (\overrightarrow{IA},\overrightarrow{IA'}) = -\frac{\pi}{4}(2\pi) \end{cases}$$

$$\iff \begin{cases} \left|\frac{z_{A'}-z_I}{z_A-z_I}\right| = \sqrt{2} \\ Arg\left(\frac{z_{A'}-z_I}{z_A-z_I}\right) = -\frac{\pi}{4}(2\pi) \end{cases}$$

$$\iff \frac{z_{A'}-z_I}{z_A-z_I} = \sqrt{2}e^{-i\frac{\pi}{4}} = 1\text{-}i$$

$$\iff z_{A'}-z_I = (1-i)(z_A-z_I)$$

$$\iff -iz_I = (1-i)(z_A) - z_{A'}$$

$$\iff -iz_I = (1-i)(3-i) - 3i\sqrt{2}$$

$$\iff -iz_I = 2-i(4+3\sqrt{2})$$

$$z_I = (4+3\sqrt{2})+2i$$

c) $f: M(z) \longrightarrow$ M'(z') tels que

$$z' = \sqrt{2}e^{-i\frac{\pi}{4}}(z-z_I)+z_I$$

$$z' = (1-i)z - 2 + i\,(4+3\sqrt{2})$$

2) $f_{(K)} = 0 \iff$

$$(1\text{-}i)z_K - 2 + i(4+3\sqrt{2}) = 0$$

$$z_K = \frac{6+3\sqrt{2}-i(2+3\sqrt{2})}{2}$$

EXERCICE 17:

1) $AC = |z_C - z_A| = |4+2i| = 2\sqrt{5}$

$BD = |z_D - z_B| = |-2+4i| = 2\sqrt{5}$

$$(\overrightarrow{AC},\overrightarrow{BD}) = Arg\left(\frac{z_D-z_B}{z_C-z_A}\right)(2\pi)$$

$$= Arg\left(\frac{-2+4i}{4+2i}\right)(2\pi)$$

$$= Arg(i)(2\pi)$$

$$= \frac{\pi}{2}\ (2\pi)$$

AC=BD et AC≠0 d'où il existe un unique déplacement r tels que r(A)=B et r(C)=D

r est d'angle $\frac{\pi}{2}$

D'où r est une rotation d'angle $\frac{\pi}{2}$

*soit w son centre

$$r(A)=B \begin{cases} wA = wB \\ (\overrightarrow{wA},\overrightarrow{wB}) = \frac{\pi}{2}\ (2\pi) \end{cases}$$

$$\iff \begin{cases} |z_B - z_w| = |z_A - z_w| \\ Arg\left(\frac{z_B-z_w}{z_A-z_w}\right) = \frac{\pi}{2}\ (2\pi) \end{cases}$$

$$\iff \begin{cases} \left|\frac{z_B-z_w}{z_A-z_w}\right| = 1 \\ Arg\left(\frac{z_B-z_w}{z_A-z_w}\right) = \frac{\pi}{2}\ (2\pi) \end{cases}$$

$$\iff \frac{z_B-z_w}{z_A-z_w} = e^{i\frac{\pi}{2}} = i$$

$$\iff z_B - z_w = i(z_A-z_w)$$

$$\iff (1\text{-}i)\,z_w = z_B - iz_A$$

$$\iff (1\text{-}i)\,z_w = 2$$

$$\iff z_w = \frac{2}{1-i} = 1+i = z_I$$

Conclusion: $r = r_{(I,\frac{\pi}{2})}$

Qarray.tn -Aqra Blech- Qarray.tn